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TECHNICAL MEMORANDUMS

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 368

# M E T A L   S P A R S

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From "Flight," February 25, 1926



Washington
June, 1926

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

TECHNICAL MEMORANDUM NO. 368.

# METAL SPARS.*

By J. D. Haddon.

It has been my experience that several firms starting metal construction are spending unnecessary money by designing their first spar section on the trial and error system. Also the man in the drawing office is at a disadvantage, through lack of experience in this type of construction, in stressing, designing rolls, etc. A lot of this could be obviated if the designer had at his disposal certain information, which it is the object of this article to supply.

Fig. 1 shows three typical rolled steel spars.

Metal spars are so varied in design that it is impossible to discuss them all here. However, certain principles are common to all. We will therefore follow the design of one throughout.

Assume a spar is required for a bay 30 in. long (l), 5000 lb. end load (P), 120 lb. per ft. run (w) and bending moments at ends 1500 ft.-lb. (MA) and 1400 ft.-lb. (MB).

The first step is to choose the form our spar is to take. In this connection the following points must be taken into account:

Edges of metal must not be at points of high stress, i.e.,

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top or sides as Fig. 2 (a and b). (b) would be O.K. if the end had a small radius as Fig. 1 (a). Do not allow any flats as in Fig. 2 (c) X - X. Two large radii adjacent produce a flat where they meet; always at highly stressed parts put a small radius next to a large, as Fig. 1 (a) flange. This is not so necessary in the web where the stress is small.

Maximum radius must not exceed 30t. It is no good taking the figure for tubes. It is obvious that an inch diameter tube will stand a greater r/t ratio than a tubular section say 12 in. diameter made up of several ½-inch radii.

Medium radius for DTD 16/50 steel is 3t and DTD 16/52 is 1t.

Neglect of any of the above will lead to local failure before the spar develops its stress.

Remember that fittings and ribs will have to be attached after the spar is made up.

I have seen spars designed with the one thought that they must stand up to the stress and then handed to the draughtsman to incorporate in a wing, with the result that most elaborate and heavy fittings have to be made and in one case rib posts had to be attached before the spar was made up. It generally pays to make the webs of a lighter gauge than the flanges.

Heat treated strip in long length coils cannot at present be had in a greater width than 6½ in. for 0.015 in. to 0.032 in. thick, and 4½ in. for 0.010 in. to 0.015 in. thick. For fittings

and side plates the steel K.E. 169 is supplied in the annealed state up to 7 in. in width.

We will make our spar of the type shown in Fig. 1 (a). Let its dimensions be as shown in Fig. 3 (a) and (b), dimensions in (b) being obtained by scaling a drawing ten times full size.

Our next procedure is to find the area and moment of inertia. Area = length × thickness.

Length of flange =

$$2\left(\frac{(50 \times 0.15) + (150 \times 0.4) + (100 \times 0.1)}{57.3} + 0.18\right) = 3.064 \text{ in.}$$

Area of flanges = 3.064 × 0.022 × 2 = 0.1348 sq.in.

Length of web =

$$2\left(\frac{(270 \times 0.1) + (70 \times 0.2) + (70 \times 0.5)}{57.3} + 0.38\right) = 3.41 \text{ in.}$$

Area of webs = 3.41 × 0.018 × 2 = 0.123 sq.in.

Total area = 0.1348 + 0.123 = 0.2578 sq.in.

The moment of inertia is best found by use of calculus.

If we take any rectangle (Fig. 4 (a)) of length $l$, breadth b, and distance d from a line xx

$$Ixx = lb\left(d^2 + \frac{b^2}{24}\right);$$

if $\frac{b}{d}$ is very small it will be sufficiently accurate to say $Ixx = lbd^2$.

Now take the arc AB (Fig. 4 (b)) of radius b and thick-

ness $t$, $t$ being very small in relation to the distance from xx. The inertia is required about xx. CD is an infinitesimally small portion of the arc and its distance from xx will be $a + b\cos\theta$ and its inertia about xx, $t(a + b\cos\theta)^2\, b\,d\theta$, i.e., distance squared × length × thickness.

The inertia of the whole arc AB will therefore be:

$$t\int_{-\beta}^{\alpha} (a + b\cos\theta)^2\, b\,d\theta$$

$$= tb\int_{-\beta}^{\alpha} (a^2 + 2ab\cos\theta + b^2\cos^2\theta)\,d\theta$$

$$= tb\int_{-\beta}^{\alpha} \left(a^2 + \frac{b^2}{2} + 2ab\cos\theta + \frac{b^2}{2}\cos 2\theta\right) d\theta.$$

$$= tb\left[\left(a^2 + \frac{b^2}{2}\right)\theta + 2ab\sin\theta + \frac{b^2}{4}\sin 2\theta\right]_{-\beta}^{\alpha}$$

Using this method for our spar we have:

$$\frac{I\ \text{flange}}{4t} = \int_{0}^{50} (1.518 - 0.15\cos\theta)^2\ 0.15\,d\theta$$

$$+ \int_{-100}^{50} (1.167 + 0.4\cos\theta)^2\ 0.4\,d\theta$$

$$+ \int_{50}^{100} (1.08 - 0.1\cos\theta)^2\ 0.1\,d\theta + 0.18 \times 0.981^2$$

$$= 0.15\left[\left(1.518^2 + \frac{0.15^2}{2}\right)\frac{50}{57.3} - 2 \times 1.518 \times 0.15 \times 0.766 + \frac{0.15^2}{4} \times 0.985\right]$$

$$+ 0.4\left[\left(1.167^2 + \frac{0.4^2}{2}\right)\frac{150}{57.3} + 2 \times 1.167 \times 0.4 \times (0.766 + 0.985) + \frac{0.4^2}{4}(0.985 - 0.342)\right]$$

$$+ 0.1 \left[\left(1.08^2 + \frac{0.1^2}{2}\right) \frac{100}{57.3} - 2 \times 1.08 \times 0.1 \times 0.985 - \frac{0.1^2}{4} \times 0.342\right] + 0.18 \times 0.981^2$$

$$= 0.251 + 2.1755 + 0.1837 + 0.1735$$

$$= 2.7827$$

$$I \text{ flange} = 2.7827 \times 4 \times 0.22 = \underline{0.245 \text{ in.}^4}$$

$$\frac{I \text{ web}}{4t} = 0.1 \times 1.057^2 \times \pi + 0.961^2 \times 0.18$$

$$+ \int_0^{90} (0.858 + 0.1 \cos\theta)^2 \; 0.1 \, d\theta$$

$$+ \int_{0.658}^{0.858} x^2 dx + \int_{20}^{90} (0.658 - 0.2 \cos\theta)^2 \; 0.2 \, d\theta$$

$$+ \int_0^{70} (0.5 \cos\theta)^2 \; 0.5 \, d\theta$$

$$= 0.35 + 0.166 + 0.1 \left[\left(0.858^2 + \frac{0.1^2}{2}\right) \frac{90}{57.3} + 2 \times 0.858 \times 0.1 \times 1\right] + \left[\frac{0.858^3}{3} - \frac{0.658^3}{3}\right]$$

$$+ 0.2 \left[\left(0.658^2 + \frac{0.2}{2}\right) \frac{70}{57.3} - 2 \times 0.658 \times 0.2 \, (1 - 0.342) - \frac{0.2^2}{4} \times 0.642\right]$$

$$+ 0.5 \left[\frac{0.5^2 \times 70}{2 \times 57.3} + \frac{0.5^2}{4} \times 0.643\right]$$

$$= 0.35 + 0.166 + .134 + 0.116 + 0.075 + 0.097 = 0.938$$

$$I \text{ web} = 0.938 \times 4 \times 0.018 = \underline{0.068 \text{ in.}^4}$$

$$\text{Total } I \text{ of spar} = 0.245 + 0.068 = \underline{0.313 \text{ in.}^4}$$

By Webb and Thorne's formula for maximum bending moment we have:

$$\text{Euler load} \quad Q = \frac{\pi^2 EI}{l^2} = \frac{\pi^2 \times 30 \times 10^6 \times 0.313}{30 \times 30} = \underline{102800}$$

$$M(\max) = \frac{Q}{Q - P} \left\{ \frac{1}{2} (MA + MB) \left[ 1 + 0.26 \frac{P}{Q} \right] \right.$$

$$\left. + 1.02 \frac{wl^2}{8} \right\} + \frac{1}{2} \frac{(MA - MB)^2}{wl^2}$$

$$= \frac{102800}{97800} \left\{ 0.5 \ (1900) \left( 1 + 0,26 \times \frac{5000}{102800} \right) \right.$$

$$\left. + 1.02 \times \frac{120 \times 2.5^2}{8} \right\} + \frac{0.5 \times (1100)^2}{120 \times 2.5^2} = \underline{1918 \text{ ft.lb.}}$$

$$f(\max) = \frac{12 \ My}{I} + \frac{P}{A} = \frac{12 \times 1918 \times 1.575}{0.313} + \frac{5000}{0.2578}$$

$$= 115700 + 19400$$

$$= 135100 \text{ lb./sq.in.}$$

Therefore, this spar will be O.K. in DTD 16/50 steel. We will now get out the rolls or dies for making the sections out of flat sheet. These must be made very accurately, for if the strip is gripped tighter in one place than another it will come out twisted.

The first rolls are got out by sight from experience; the last but one is generally made of similar dimensions to the finished section and the last has to be such that the spring back will give the correct section, and is arrived at in the following manner:

We know that $\frac{E}{R} = \frac{f}{y}$.

If we denote the finished section constants by $E_1$ $R_1$ $f_1$ and $y_1$ and those of the section in the last die by $E_2$ $R_2$ $f_2$ and $y_2$ we get $\frac{E_2}{R_2} - \frac{E_1}{R_1} = \frac{f_2}{y_2} - \frac{f_1}{y_1}$, but $f_1 = 0$, as there is no stress in the metal after spring back and $f_2$ = yield stress, as the material yields in the die to give it its new curvature. This is not the yield stress from specification, but from test of material used, and is about 70 tons per square inch in DTD 16/50.

$$E_1 = E_2 \text{ and } y_1 = y_2 .$$

We can therefore write $E\left(\frac{1}{R_2} - \frac{1}{R_1}\right) = \frac{f_2}{y}$

and $\frac{1}{R_2} - \frac{1}{K_1} = \frac{f_2}{Ey}$ .

We will work out the last die for the flange. Let us denote the arc of .15" radius by 'a,' .4 by 'b,' and .1 by 'c'

then $$\frac{1}{Ra} = \frac{70}{13600 \times 0.011} + \frac{1}{0.15} = \frac{1}{2.137} + \frac{1}{0.15} = \frac{1}{0.1395}$$

$$\frac{1}{Rb} = \frac{1}{2.137} + \frac{1}{0.4} = \frac{1}{0.3365}$$

$$\frac{1}{Rc} = \frac{1}{2.137} + \frac{1}{0.1} = \frac{1}{0.0957}$$

This gives our radii for the last die as .1395", .3365" and .0957" respectively. Now the length of arc of each separate curve must remain the same for all the rolls and dies.

Therefore the new angles will be

$$(a) \quad \frac{0.15 \times 50}{0.1395} = 53.7^{\circ}$$

$$(b) \quad \frac{0.4 \times 150}{0.3365} = 178.7^{\circ}$$

$$(c) \quad \frac{0.1 \times 100}{0.0957} = 104.3^{\circ}$$

Fig. 5 shows the rolls and dies. The first two may be rolls but the last two must be dies; the distance x being less than y the rolls could not run in each other. No. 3 could be made suitable for rolls by cutting away the small portion that fouls. This being so small would not harm the strip.

The axes of the male and female rolls should be equidistant from the center of the strip x - x; this minimizes the amount of slip caused by the difference in linear velocity of the rolls, which increased with their difference of diameter at any point.

The rolls for the webs may be found in the same way; it will be better to have five instead of four for this section.

Some firms have fitted a furnace to their drawbench and they draw the metal hot.

I have no experience of results, but would imagine that the dies would be difficult to design, the yield point being an unknown quantity. Against this method is the very much

greater time taken. With good tools and a little experience, good results should be had from drawing cold. Twist is often due to badly made dies or drawing through two at once that are not in line.

It is better to have the rolls sliding on a splined shaft; if they are fixed it is very difficult to get three or four dead in line. Always have a lead in on the dies. Make, say, 1/4 inch of the die the true section and chamfer the rest off.

A test should be made of all new spars to see that they develop the required stress without local buckle.

FIG. 1

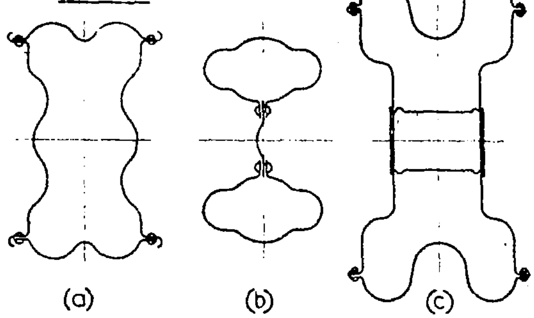
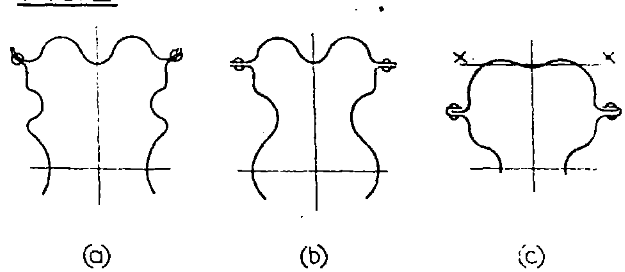



FIG. 2



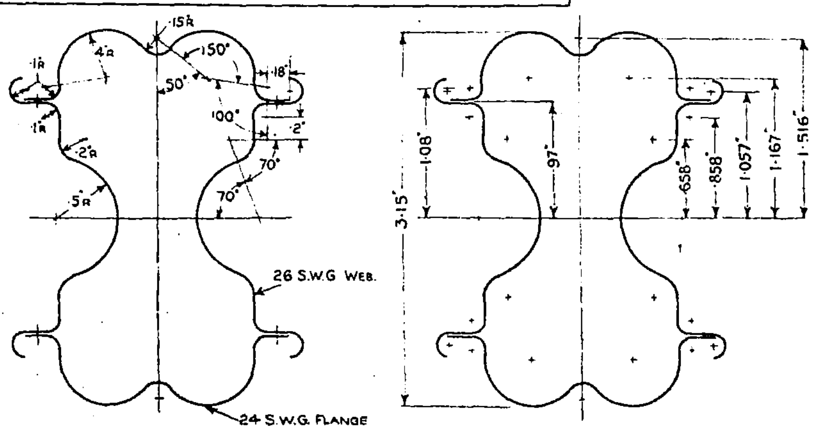




FIG.3

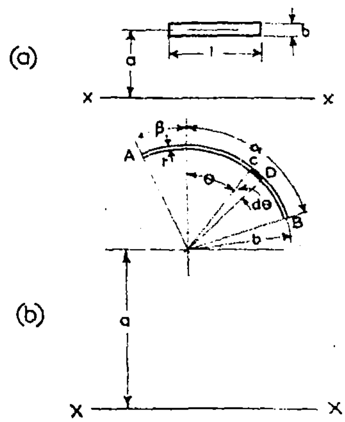



FIG. 4

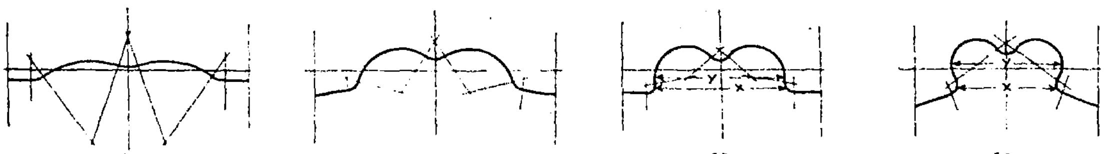



FIG. 5